Writing CFT correlation functions as AdS scattering amplitudes

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Outline

- Introduction
- Mellin amplitudes
- Flat space limit of AdS
- Open questions

Introduction

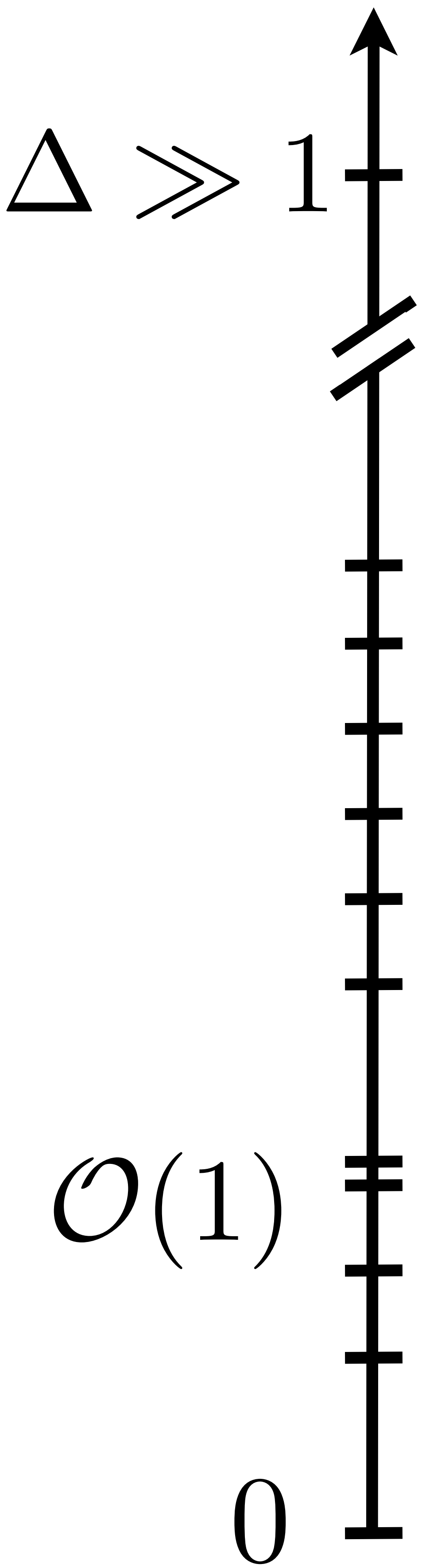
The AdS description of a CFT is useful if it is

- **Weakly coupled** $\longleftrightarrow$ "large N" CFT
- **Local** - effective field theory in AdS with small number of fields valid up to some UV cutoff ℓ much smaller than the AdS radius R

$$\ell \sim \frac{1}{\mathrm{mass}} \sim \frac{R}{\Delta}$$

$\rightarrow$ Large gap in spectrum of dimensions $\quad \Delta \gg 1$

Spectrum of Effective CFT



Other single-trace

AdS coupling constant

Multi-trace primaries

$$\dim\left(\mathcal{O}_1 \partial^n \mathcal{O}_2\right) = \Delta_1 + \Delta_2 + n + \mathcal{O}\left(1/N^2\right)$$

Few single-trace primaries spin ≤ 2

Unitary bound

[Heemskerk, JP, Polchinski, Sully '09]

[Fitzpatrick, Katz, Poland, Simmons-Duffin '10]

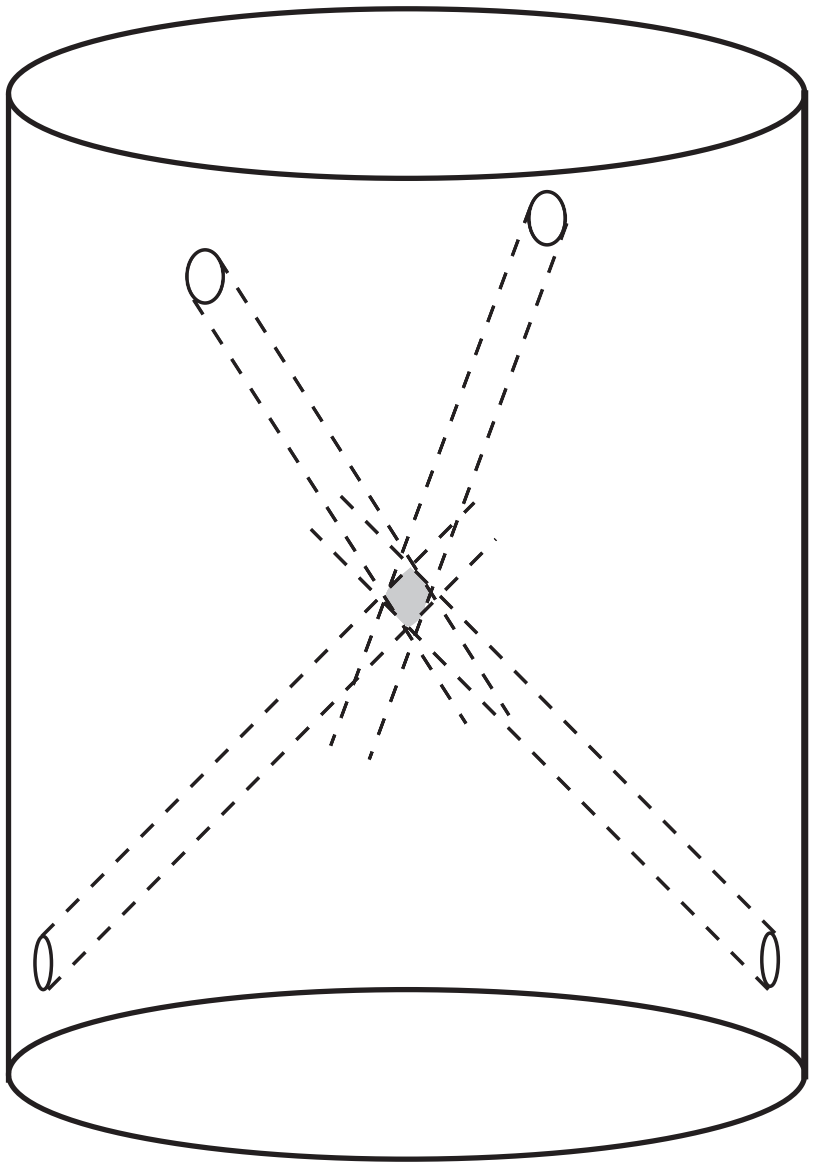
Bulk Locality

Main difficulty: local bulk physics is encoded in CFT correlation functions in a non-trivial way.

How to extract the bulk S-matrix?

Idea: prepare wave-packets that scatter in small region of AdS

[Polchinski 99]
[Susskind 99]
[Gary, Giddings, JP '09]
[Okuda, JP '10]



Best language: Mellin amplitudes

Mellin Amplitudes

[Mack '09]

Correlation function of scalar primary operators

$$A(x_i) = \langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle$$

$$A(x_i) = \mathcal{N} \int_{-i\infty}^{i\infty} [d\delta] \boxed{M(\delta_{ij})} \prod_{i<j}^{n} \Gamma(\delta_{ij}) \left(x_{ij}^2\right)^{-\delta_{ij}}$$

Constraint $\quad \sum_{j \neq i}^{n} \delta_{ij} = \Delta_i = \dim[\mathcal{O}_i]$

integration variables = # ind. cross-ratios = $\dfrac{n(n-3)}{2}$

Analogy with Scattering Amplitudes

Introduce k_i such that $-k_i^2 = \Delta_i$ and $\sum_{i=1}^{n} k_i = 0$

then $\delta_{ij} = k_i \cdot k_j$ automatically solves the constraints.

Define $s_{ij} = -(k_i + k_j)^2 = \Delta_i + \Delta_j - 2\delta_{ij}$ [Mack '09]

The Mellin amplitude is crossing symmetric and meromorphic with simple poles at ($n = 4$)

$$M(s_{ij}) \approx \frac{C_{13k} C_{24k} P_{l_k}(\gamma_{13})}{s_{13} - (\Delta_k - l_k + 2m)} \qquad m = 0, 1, 2, \ldots$$

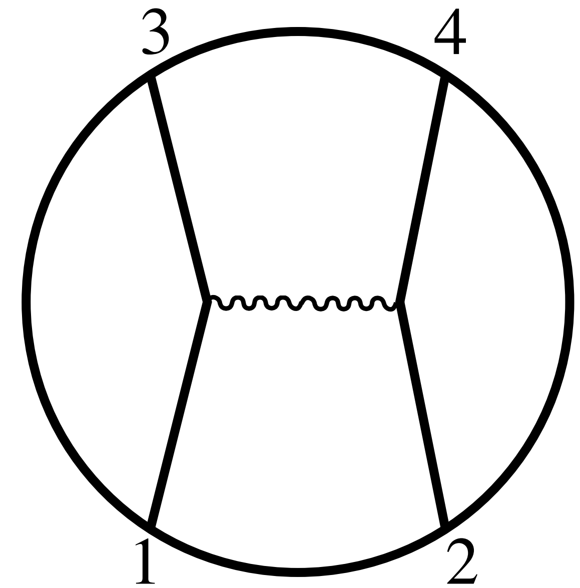
$$\mathcal{O}_1 \mathcal{O}_3 \sim C_{13k} \mathcal{O}_k$$
$$\mathcal{O}_2 \mathcal{O}_4 \sim C_{24k} \mathcal{O}_k$$

$$\gamma_{13} = \frac{1}{2}(s_{12} - s_{14})$$

Example: Graviton exchange in AdS5

Minimally coupled massless scalars

$$\Delta_i = d = 4$$



[D'Hoker, Freedman, Mathur, Mathusis, Rastelli '99]

$$A(x_i) \propto 9D_{4444}(x_i) - \frac{4}{3x_{13}^6} D_{1414}(x_i) - \frac{20}{9x_{13}^4} D_{2424}(x_i) - \frac{23}{9x_{13}^2} D_{3434}(x_i)$$
$$+ \frac{16(x_{14}^2 x_{23}^2 + x_{12}^2 x_{34}^2)}{3x_{13}^6} D_{2525}(x_i) + \frac{64(x_{14}^2 x_{23}^2 + x_{12}^2 x_{34}^2)}{9x_{13}^4} D_{3535}(x_i)$$
$$+ \frac{8(x_{14}^2 x_{23}^2 + x_{12}^2 x_{34}^2 - x_{24}^2 x_{13}^2)}{x_{13}^2} D_{4545}(x_i) \ .$$

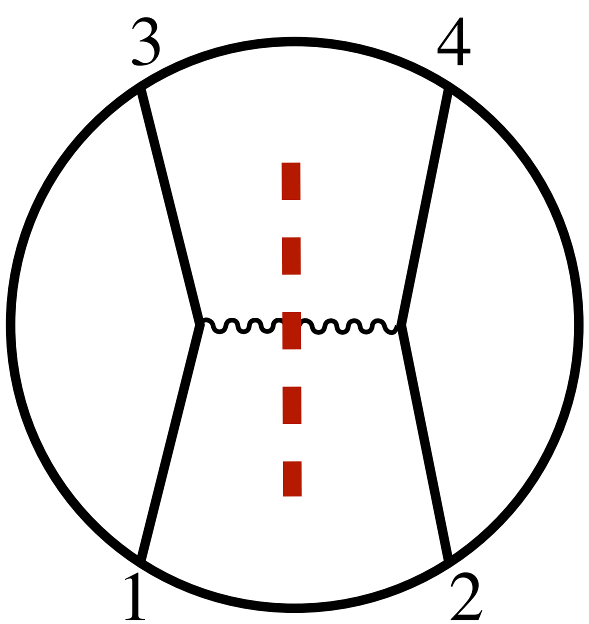
D-function

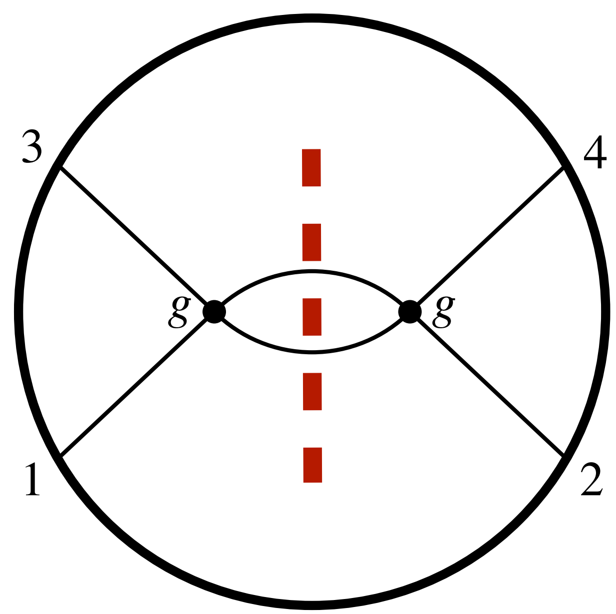
$$M(s_{ij}) \propto \frac{6\gamma_{13}^2 + 2}{s_{13} - 2} + \frac{8\gamma_{13}^2}{s_{13} - 4} + \frac{\gamma_{13}^2 - 1}{s_{13} - 6} - \frac{15}{4} s_{13} + \frac{55}{2}$$

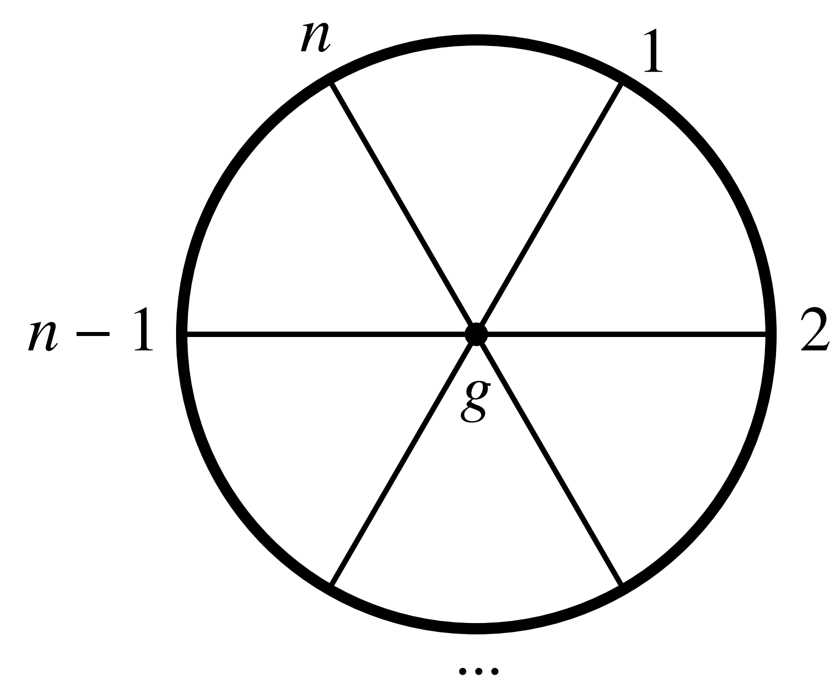
Double-trace operators

The double-trace operators $\mathcal{O}_i \partial^n \mathcal{O}_j$ (normal ordered product of external operators) do **not** give rise to poles in the Mellin amplitude.

All poles are associated with on-shell internal states.





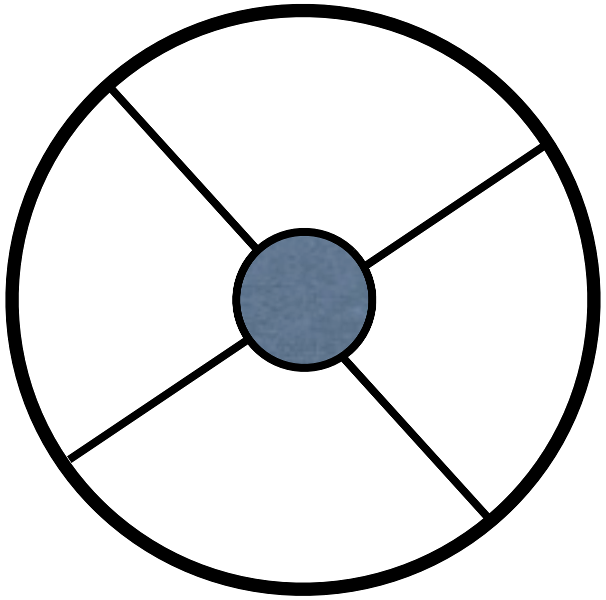


Contact diagrams in AdS give polynomial Mellin amplitudes

Mellin amplitudes are specially nice in planar CFT's (dual to tree level string theory in AdS).

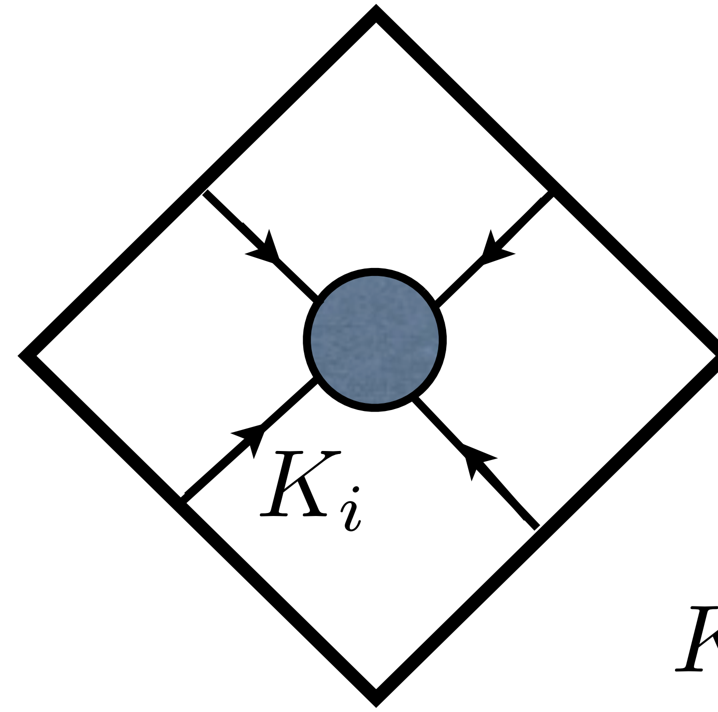
Flat space limit of AdS

Anti-de Sitter



$R \to \infty$

Minkowski



$K_i^2 = 0$

$$M_i^2 = \frac{\Delta_i(\Delta_i - d)}{R^2}$$

$$S_{ij} = -(K_i + K_j)^2$$

$$M(s_{ij}) \approx \frac{R^{n(1-d)/2+d+1}}{\Gamma\left(\frac{1}{2}\sum_i \Delta_i - \frac{d}{2}\right)} \int_0^\infty d\beta\, \beta^{\frac{1}{2}\sum_i \Delta_i - \frac{d}{2} - 1} e^{-\beta}\, T\left(S_{ij} = \frac{2\beta}{R^2} s_{ij}\right)$$

Mellin amplitude for $s_{ij} \gg 1$

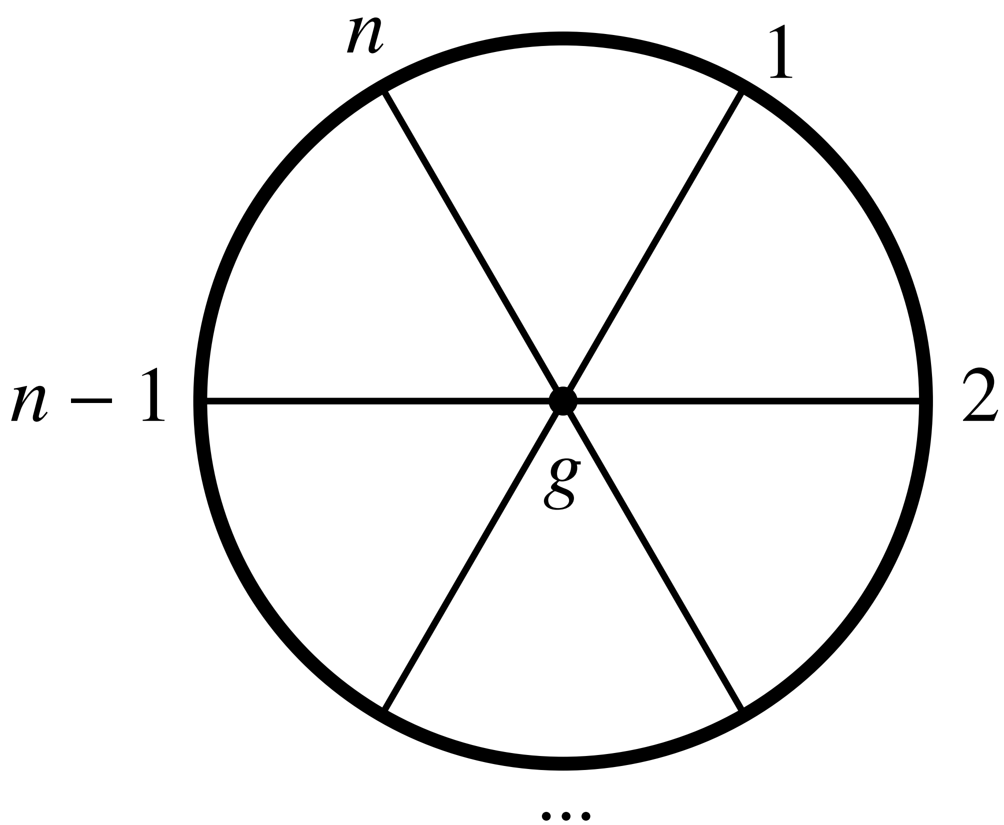
Scattering amplitude

Evidence for $M \approx \int \ldots T$

1) Works for an infinite set of interactions

$$g\nabla\ldots\nabla\phi_1\nabla\ldots\nabla\phi_2\ldots\nabla\ldots\nabla\phi_n$$

α_{12} contractions



$$\#\ \text{derivatives} = 2\sum_{i<j}^{n}\alpha_{ij} = 2N$$

$$T(S_{ij}) = g\prod_{i<j}^{n}\left(\frac{S_{ij}}{2}\right)^{\alpha_{ij}}$$

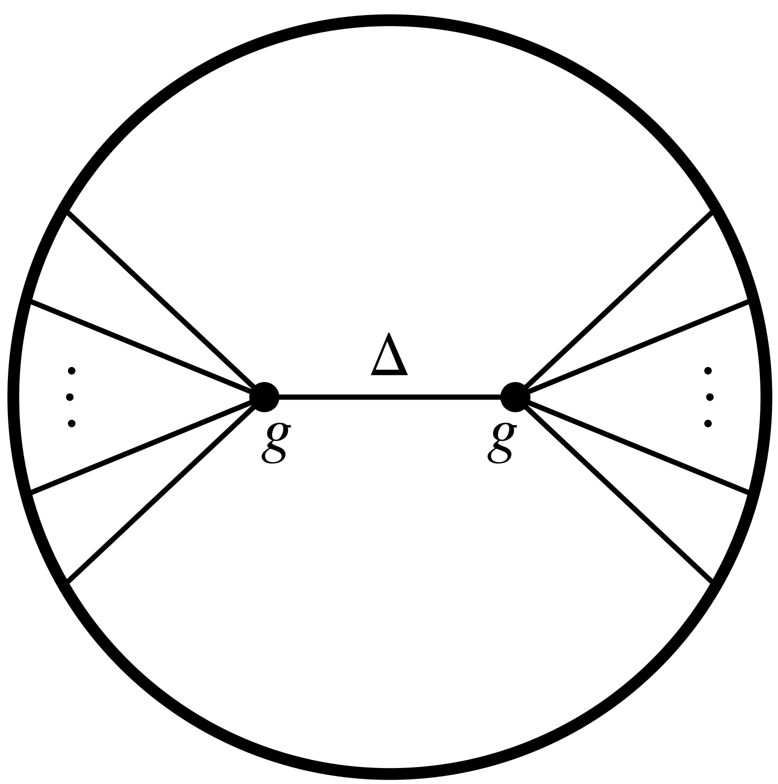
$$M(s_{ij}) \approx \underbrace{gR^{n(1-d)/2+d+1-2N}}_{\text{dimensionless}}\frac{\Gamma\left(\frac{1}{2}\sum_i\Delta_i - \frac{d}{2} + N\right)}{\Gamma\left(\frac{1}{2}\sum_i\Delta_i - \frac{d}{2}\right)}\prod_{i<j}^{n}(s_{ij})^{\alpha_{ij}}$$

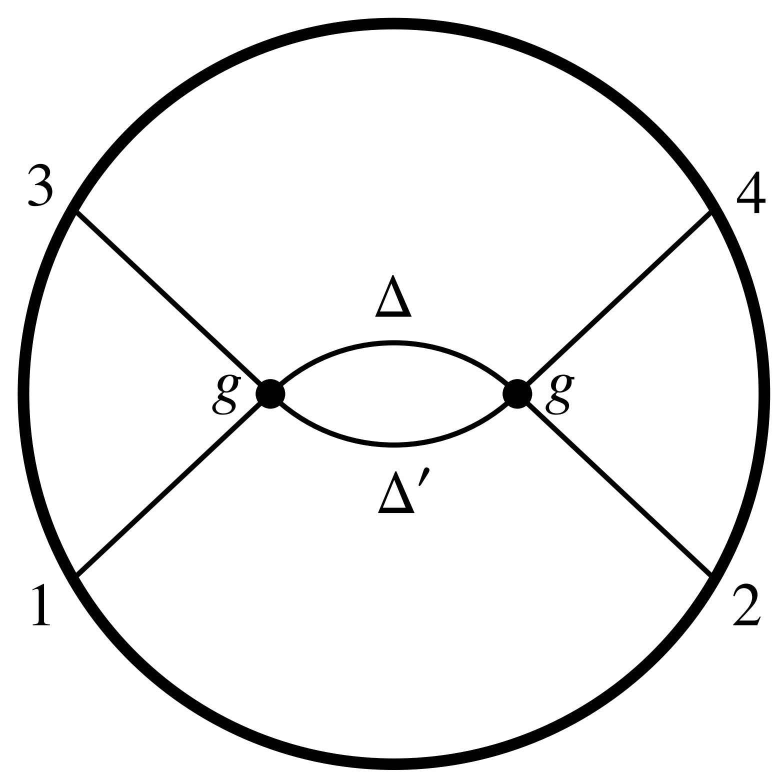
Evidence for $M \approx \int \ldots T$

2) Agrees with previous results based on wave-packet constructions

[Gary, Giddings, JP '09]
[Okuda, JP '10]

3) Works in several non-trivial examples





Application: from SYM to IIB strings

$\mathcal{N} = 4$ SYM $\longleftrightarrow$ type IIB strings

$$g_{\rm YM}^2 = 4\pi g_s$$
$$g_{\rm YM}^2 N = \lambda = (R/\ell_s)^4$$

$\mathcal{O}(x) =$ Lagrangian density

$\phi =$ Dilaton

4pt function
$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$

$\longrightarrow$ $R \to \infty$

2 → 2 scattering amplitude

$$\lim_{\lambda\to\infty} \lambda^{\frac{3}{2}} M(g_{\rm YM}^2, \lambda, s_{ij} = \sqrt{\lambda}\alpha_{ij}) = \frac{1}{120\pi^3 \ell_s^6} \int_0^\infty d\beta \beta^5 e^{-\beta} T_{10}\left(g_s, \ell_s, S_{ij} = \frac{2\beta}{\ell_s^2}\alpha_{ij}\right)$$

Mellin amplitude

Open Questions

- Generalize to external massive particles (work in progress)
  ⟶ 3pt-functions of SYM at strong coupling
- Mellin amplitudes for external operators with spin (helicity)
- Build n-pt functions by "gluing" 3pt functions of single-trace operators (analogous to BCFW - Suvrat's talk)
- Feynman rules for Mellin amplitudes?
- Unitarity for Mellin amplitudes? [Fitzpatrick, Katz, Poland, Simmons-Duffin '10]
  Renormalizable vs non-renormalizable bulk interactions
- Bootstrap for CFT in higher dimensions (d>2)
- Mellin amplitudes without conformal invariance?